Trig Identity Problems

Problem Simplify $\dfrac{\sin(20^\circ)}{\cos(10^\circ)}$

We have $\dfrac{\sin(20^\circ)}{\cos(10^\circ)} = \dfrac{2\sin(10^\circ)\cos(10^\circ)}{\cos(10^\circ)}$

$= 2\sin(10^\circ)$

Problem Evaluate $\tan(10^\circ) \times \tan(20^\circ) \times \cdots \times \tan(80^\circ)$

We have

$\tan(10^\circ) \times \tan(20^\circ) \times \cdots \times \tan(80^\circ)$

$$= \frac{\sin(10^\circ)}{\cos(10^\circ)} \times \frac{\sin(20^\circ)}{\cos(20^\circ)} \times \cdots \times \frac{\sin(80^\circ)}{\cos(80^\circ)} \qquad (*)$$

Note that $\sin(x) = \cos(90 - x)$

We rewrite $(*)$ as

$$\frac{\sin(10^\circ)}{\cos(80^\circ)} \times \frac{\sin(20^\circ)}{\cos(70^\circ)} \times \cdots \times \frac{\sin(80^\circ)}{\cos(10^\circ)}$$

$= 1 \times 1 \times \cdots \times 1 = 1$

Problem Write

$[\sin(13°)+\sin(167°)+\cos(13°)+\cos(167°)]$

$\times[\sin(13°)-\sin(167°)+\cos(13°)-\cos(167°)]$

in the Form $a\sin(x°)$

Note that $13+167=180$

We will apply the identities

$$\sin(180-x) = \sin(x)$$

and $$\cos(180-x) = -\cos(x)$$

So the product can be rewritten as

$[\sin(13°)+\sin(13°)+\cos(13°)-\cos(13°)]$

$\times[\sin(13°)-\sin(13°)+\cos(13°)+\cos(13°)]$

$= [2\sin(13°)][2\cos(13°)]$

$= 2\sin(26°)$

Find x if $\operatorname{Tan}^{-1}(x) = \operatorname{Tan}^{-1}(4)+\operatorname{Tan}^{-1}(6)$

[Note - Tan^{-1} denotes the principal value]

We apply tan to both sides to obtain

$$\tan(\operatorname{Tan}^{-1}(x)) = \tan(\operatorname{Tan}^{-1}(4)+\operatorname{Tan}^{-1}(6))$$

$$\Rightarrow \quad x = \frac{\tan(\mathrm{Tan}^{-1}(4)) + \tan(\mathrm{Tan}^{-1}(6))}{1 - \tan(\mathrm{Tan}^{-1}(4))\tan(\mathrm{Tan}^{-1}(6))}$$

Using the result $\tan(x+y) = \dfrac{\tan(x) + \tan(y)}{1 - \tan(x)\tan(y)}$

$$\therefore \quad x = \frac{4+6}{1-4\times 6} = \frac{10}{1-24} = \frac{-10}{23}$$

Given that $\sin(x) + \cos(x) = \sqrt{\frac{3}{2}}$ and $x \in [0, \pi)$, find x

We have $(\sin(x) + \cos(x))^2 = \frac{3}{2}$

$$\Rightarrow \quad \sin^2(x) + 2\sin(x)\cos(x) + \cos^2(x) = \frac{3}{2}$$

$$\Rightarrow \quad 1 + \sin(2x) = \frac{3}{2}$$

$$\Rightarrow \quad \sin(2x) = \frac{3}{2} - 1 = \frac{1}{2}$$

$$\Rightarrow \quad 2x = \sin^{-1}\left(\frac{1}{2}\right)$$

$$\Rightarrow \quad x = \frac{1}{2}\sin^{-1}\left(\frac{1}{2}\right)$$

$$= \frac{1}{2}\cdot\frac{\pi}{6}, \quad \frac{1}{2}\cdot\frac{7\pi}{6}$$

$$= \frac{\pi}{12}, \quad \frac{7\pi}{12}$$

Note: given a value for $\sin(x) \pm \cos(x)$, we can use the procedure above to find x

Find the value of

$$\sin^2(10^\circ) + \sin^2(20^\circ) + \cdots + \sin^2(90^\circ)$$

We have $\sin(x) = \cos(90 - x)$

so $\sin(10) = \cos(80)$

$\sin(20) = \cos(70)$

$\sin(30) = \cos(60)$

$\sin(40) = \cos(50)$

$\therefore \sin^2(10) + \cdots + \sin^2(90)$

$= \cos^2(80) + \cos^2(70) + \cos^2(60) + \cos^2(50)$

$+ \sin^2(50) + \sin^2(60) + \sin^2(70) + \sin^2(80)$

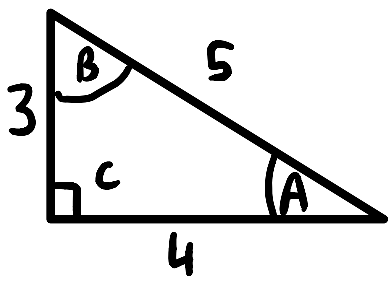
$+ \sin^2(90)$

$= 1 + 1 + 1 + 1 + 1$ (since $\sin(90^\circ) = 1$)

$= 5$

Given the triangle ABC with side a opposite $\angle A$ etc, Find $\sin(A) + \sin(2B) + \sin(3C)$ if $a = 3$, $b = 4$, $c = 5$.

A 3-4-5 triangle is right-angled:



We have $\sin(A) = \frac{3}{5}$

Since $C = \frac{\pi}{2}$, we have

$3C = \frac{3\pi}{2}$ and so $\sin(3C) = -1$.

Finally,
$$\begin{aligned}\sin(2B) &= 2\sin(B)\cos(B) \\ &= 2 \cdot \left(\frac{4}{5}\right) \cdot \left(\frac{3}{5}\right) \\ &= \frac{24}{25}\end{aligned}$$

So
$$\begin{aligned}\sin(A) + \sin(2B) + \sin(3C) &= \frac{3}{5} + \frac{24}{25} - 1 \\ &= \frac{14}{25}\end{aligned}$$