For all real numbers x except 0 and 1, the Function f is defined by

$$f\left(\frac{x}{x-1}\right) = \frac{1}{x}$$

Suppose $0 < \theta < \frac{\pi}{2}$

What is $f(\sec^2\theta)$?

Introduce a new variable

$$y = \frac{x}{x-1}$$

solve For x : $y(x-1) = x$

$$\Rightarrow yx - y = x$$

$$\Rightarrow yx - x = y$$

$$\Rightarrow x(y-1) = y$$

$$\Rightarrow x = \frac{y}{y-1} \qquad (y \neq 1)$$

IF $y = 0$ then $x = 0$ which is not possible

so for $y \neq 0, 1$ we have

$$f(y) = f\left(\frac{x}{x-1}\right)$$

$$= \frac{1}{x}$$

$$= \frac{1}{\left(\frac{y}{y-1}\right)}$$

$$= \frac{y-1}{y}$$

$$= 1 - \frac{1}{y}$$

When $y = \sec^2\theta$, we have

$$f(\sec^2\theta) = 1 - \frac{1}{\sec^2\theta}$$

$$= 1 - \cos^2\theta$$

$$= \sin^2\theta$$