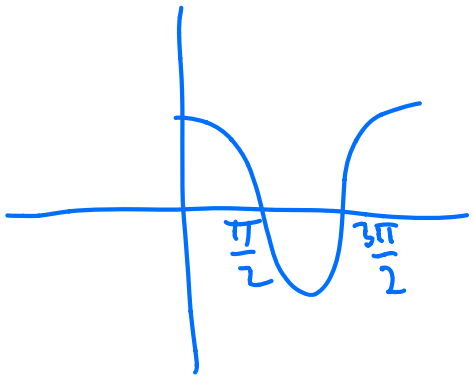
How many zeroes does $f(x) = \cos(\log(x))$ have on the interval $0 < x < 1$?

For $0 < x < 1$, we have $-\infty < \log(x) < 0$

Now, $\cos(y) = 0 \iff y = \frac{\pi}{2} + n\pi$ for $n \in \mathbb{Z}$

There are infinitely many such $y \in (-\infty, 0)$ [those corresponding to $n \leq -1$] and therefore there are infinitely many zeroes of $F(x)$ in the given interval.

$\cos(y) = 0 \iff y = (2n+1)\frac{\pi}{2} \quad (n \in \mathbb{Z})$



$= \frac{2n\pi}{2} + \frac{\pi}{2}$

$= n\pi + \frac{\pi}{2}$

$\log(x) = (2n+1)\frac{\pi}{2}$

$\Rightarrow x = e^{(2n+1)\frac{\pi}{2}}$

When $n \leq -1$, $0 < x < 1$ is a soln

$\Rightarrow$ Infinitely many