Trig Half Angle Formulae

Thm $\sin\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1-\cos(\theta)}{2}}$

We have

$$\cos(\theta) = 1 - 2\sin^2\left(\frac{\theta}{2}\right)$$

$$\Rightarrow \cos(\theta) - 1 = -2\sin^2\left(\frac{\theta}{2}\right)$$

$$\Rightarrow 1 - \cos(\theta) = 2\sin^2\frac{\theta}{2}$$

$$\Rightarrow \frac{1-\cos\theta}{2} = \sin^2\frac{\theta}{2}$$

$$\Rightarrow \sin\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1-\cos\theta}{2}}$$

Thm $\cos\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1-\cos\theta}{2}}$

We have

$$\cos(\theta) = 2\cos^2\left(\frac{\theta}{2}\right) - 1$$

$$\Rightarrow \frac{\cos(\theta)+1}{2} = \cos^2\left(\frac{\theta}{2}\right)$$

$$\Rightarrow \cos\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{\cos(\theta)+1}{2}}$$

Thm $\tan\left(\frac{\theta}{2}\right) = \frac{1-\cos(\theta)}{\sin(\theta)}$

we have

$$\tan\left(\frac{\theta}{2}\right) = \frac{\sin\left(\frac{\theta}{2}\right)}{\cos\left(\frac{\theta}{2}\right)}$$

$$= \frac{\pm\sqrt{\frac{1-\cos(\theta)}{2}}}{\pm\sqrt{\frac{1+\cos(\theta)}{2}}}$$

$$= \sqrt{\frac{1-\cos\theta}{2}\cdot\frac{2}{1+\cos\theta}}$$

(4)

$$= \sqrt{\frac{1-\cos\theta}{1+\cos\theta} \cdot \frac{1+\cos\theta}{1+\cos\theta}}$$

$$= \sqrt{\frac{1-\cos^2\theta}{(1+\cos\theta)^2}}$$

$$= \sqrt{\frac{\sin^2\theta}{(1+\cos\theta)^2}}$$

$$= \frac{\sin\theta}{1+\cos\theta}$$

OR from (*):

$$\tan\left(\frac{\theta}{2}\right) = \sqrt{\frac{1-\cos\theta}{1+\cos\theta} \quad \frac{1-\cos\theta}{1-\cos\theta}}$$

$$= \sqrt{\frac{(1-\cos\theta)^2}{}}$$

$$\sqrt{1-\cos^2\theta}$$

$$= \sqrt{\frac{(1-\cos\theta)^2}{\sin^2\theta}}$$

$$= \frac{1-\cos\theta}{\sin\theta}$$

Applications: 15°